Math 1131 Week 8 Worksheet

Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

4.1: Maximum and Minimum Values

1. For the following functions, find all critical numbers **exactly**.

   (a) $f(x) = x^5 - 2x^3$

   (b) $f(x) = x - 2\sin x$ for $0 < x < 2\pi$

   (c) $f(x) = e^{-x} - e^{-3x}$ for $x > 0$

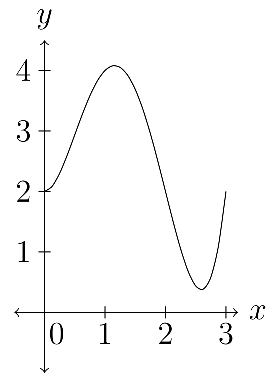
2. Use calculus to find the absolute maximum and minimum values of the following functions on the given intervals. Give your answers **exactly** and show supporting work.

   (a) $f(x) = x^3 - 2x^2 + x + 1$ on $[0, 1]$

   (b) $x^{1/3}(x - 2)$ on $[-1, 3]$

   (c) $f(x) = (7x - 1)e^{-2x}$ on $[0, 1]$

3. Below is the graph of $f(x) = x^4 - 5x^3 + 6x^2 + 2$. On the interval $[0, 3]$ determine the maximum and minimum value of the *slope* of the graph, *i.e.*, the maximum and minimum values of $g(x) = f'(x)$.



4. T/F (with justification) If $f(x)$ is a differentiable function on (a, b) and $f(x)$ has a local maximum or minimum value at $x = c$ in (a, b) then $f'(c) = 0$.

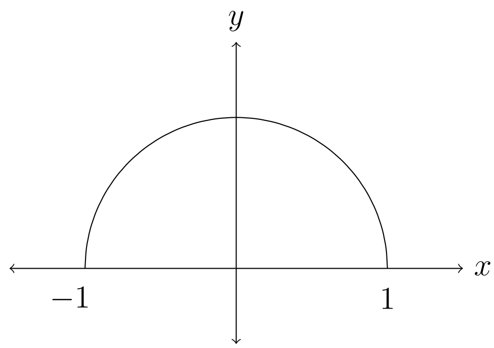
5. T/F (with justification) If $f(x)$ is a differentiable function on (a, b) and $f'(c) = 0$ for a number c in (a, b) then $f(x)$ has a local maximum or minimum value at $x = c$.

4.2: Mean Value Theorem

6. Find every number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = x^3 - 4x^2 + x$ on the interval $[0, 1]$.

7. T/F (with justification) The function $1 - \dfrac{1}{x^4}$ satisfies the hypotheses of Rolle's Theorem on the interval $[-1, 1]$.

8. T/F (with justification) The graph of the semicircle on $[-1, 1]$ below fits the hypotheses of the Mean Value Theorem.



Answers to Selected Problems:

1. (a) 0 and $\pm\sqrt{6/5}$
   (b) $x = \pi/3$ and $x = 5\pi/3$
   (c) $x = \frac{1}{2}\ln 3$

2. (a) On $[0, 1]$ the absolute maximum value of $f(x)$ is $31/27$ and the absolute minimum value of $f(x)$ is 1.
   (b) On $[-1, 3]$, the absolute maximum value of $f(x)$ is 3 and the absolute minimum value of $f(x)$ is $-(3/2)/2^{1/3} = -3/2^{4/3}$.
   (c) The absolute maximum value of $f(x)$ is $(7/2)e^{-9/7}$, and the absolute minimum value of $f(x)$ is -1.

3. $f'(x) = g(x)$ has maximum value 9 and minimum value -4.

4. True

5. False

6. $c = (8 - \sqrt{28})/6$

7. False

8. True