

Math 2141 - Section 1 - Advanced Calculus
I.2 Sets Activity

1. If both $A \subseteq B$ and $B \subseteq A$, what can we say?

Important Note:

Section I.2.4: Unions Intersections and Complements

Let A and B be sets.

Definition 1. *The union of A and B , denoted $A \cup B$ and read A union B , is*

$$A \cup B = \{x | x \in A \text{ or } x \in B\}.$$

Definition 2. *The intersection of A and B , denoted $A \cap B$ and read A intersect B , is*

$$A \cap B = \{x | x \in A \text{ and } x \in B\}.$$

Definition 3. *The sets A and B are disjoint if $A \cap B = \emptyset$.*

Definition 4. *The set difference of A and B , denoted $A - B$ or $A \setminus B$ and read A minus B , is*

$$A - B = \{x | x \in A \text{ and } x \notin B\}.$$

2. Read over the definitions above.

3. Draw a general Venn Diagram representing two intersecting sets that are not equal. Indicate which region represents $A \cup B$, $A \cap B$ and $A - B$. What does it look like to have a Venn Diagram of disjoint sets?

4. Prove $A \subseteq A \cup B$. When do we get equality?

5. Prove $A \cap B \subseteq A$. When do we get equality?

When we have a set of sets, this is usually called a class or a family. It is a set where each member is also a set. These are usually denoted by script letters let $\mathcal{A}, \mathcal{B}, \mathcal{C}$, etc.

6. What does this set represent? There are a few things that appear below we haven't mentioned yet. What do you think they mean?

$$\mathcal{A} = \left\{ \left[\frac{1}{n}, 1 \right] : n \in \mathbb{Z}^+ \right\}$$

7. Just like we can use Σ notation to add up a bunch of numbers, we can use \cap and \cup notation to take the union or intersection of all the sets in a class or family of sets. For example, we could write

$$\bigcup_{A \in \mathcal{A}} A \quad \text{or} \quad \bigcap_{A \in \mathcal{A}} A.$$

What do these mean?

8. What is

$$\bigcup_{A \in \mathcal{A}} A \quad \text{when } \mathcal{A} = \left\{ \left[\frac{1}{n}, 1 \right] : n \in \mathbb{Z}^+ \right\}?$$

9. What is

$$\bigcap_{A \in \mathcal{A}} A \quad \text{when } \mathcal{A} = \left\{ \left[\frac{1}{n}, 1 \right] : n \in \mathbb{Z}^+ \right\}?$$

10. What is

$$\bigcup_{A \in \mathcal{A}} A \quad \text{when } \mathcal{A} = \left\{ \left(\frac{1}{n}, 1 \right) : n \in \mathbb{Z}^+ \right\}?$$

11. What is

$$\bigcap_{A \in \mathcal{A}} A \quad \text{when } \mathcal{A} = \left\{ \left(\frac{1}{n}, 1 \right) : n \in \mathbb{Z}^+ \right\}?$$

12. What if we only considered $n \geq 2$?